

Quasiperiodicity in Long RF-Biased Josephson Junctions

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We present and discuss numerical simulations, using a modified sine-Gordon equation, of long Josephson junctions in a magnetic field and in the presence of rf radiation. Quasiperiodicity arises from the coexistence of the frequency of the driving force and the natural frequency of an oscillating spatial structure of soliton-like character. The quasiperiodic regime presents subharmonic response at either of those frequencies.

1. INTRODUCTION

The Josephson junction has proven to be quite useful in the study of the various routes to chaos in dynamical systems as well as the nature of chaotic behavior. In fact, few dynamical systems exhibit as many routes to chaos as the Josephson junction with no spatial extent does (1), such as the transition to chaos via period doubling, intermittency, crises and quasiperiodicity (2). Even richer behavior can be obtained in long Josephson junctions, which can also support solitonic solutions and its dynamics can have both spatial and temporal modes.

In this paper we present simulations on long Josephson junctions in the presence of an rf drive and an applied magnetic field. We show that this system exhibits a two-frequency quasiperiodic transition to chaos, which arises from the interplay between the natural frequency of the soliton, which can be excited in this system, and the external drive. This is different from the usual quasiperiodic route observed in the presence of two incommensurate driving frequencies in small Josephson junctions. In fact, the quasiperiodic transition observed in long junctions arises from the interplay between the spatial mode and the rf drive making it analogous to the quasiperiodic transition observed in Rayleigh-Bénard convection in fluids.

2. THE PHYSICAL MODEL AND ITS SIMULATION

A Josephson junction is considered to be long when its length is greater than the Josephson penetration depth λ_J . We model such a long junction with the usual sine-Gordon like equation (3).

$$\phi_{xx} - \phi_{tt} - \sin\phi = \alpha\phi_t - \rho\sin(\Omega_d t) \quad (1)$$

$$\phi_x(0, t) = \phi_x(L, t) = \eta \quad (2)$$

where ϕ is the phase difference of the superconducting order parameter between each side of the barrier and its derivative in time is the voltage across the junction. In eqn. (1) the distance is normalized to the Josephson penetration depth, time is normalized to the inverse of the Josephson plasma frequency and the rf amplitude ρ is normalized to the maximum critical current. The term $\alpha\phi_t$ represents quasiparticle loss. The constant η is a measure of the external magnetic field. We use realistic values for the different parameters which in this paper are $L=5$, $\alpha=0.252$, $\eta=1.25$, $\Omega=0.65$. We integrated eqn. (1) from flat initial conditions using a standard implicit finite-difference algorithm and plot parameters

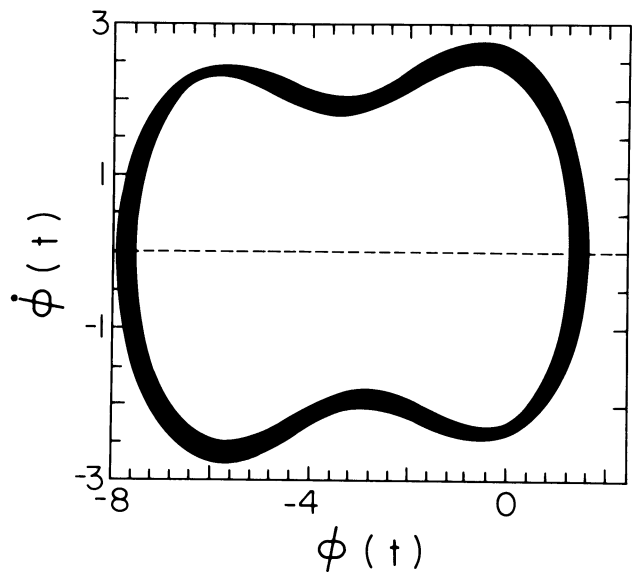


Fig.1. Phase space $\phi_t(t)$ vs $\phi(t)$: trapped quasiperiodic solution in two wells of the potential.

at the center of the junction.

3. QUASIPERIODIC TRANSITION TO CHAOS

Above the threshold of energy required for the creation of kinks or fluxons, the system is chaotic and exhibits a finite average voltage because solutions are free-running: They diffuse from one well to another and correspond to propagation of excitations along the barrier. Dissipative collisions of fluxons generate a mixture of coherent and incoherent excitations, nevertheless the motion is still governed by a strange attractor (4) despite the many degrees of freedom involved; this chaotic regime is an example of higher dimensional temporal chaos. Breather solutions are also possible in this regime: collisions between fluxons eventually lead to their annihilation (5) into a bound state and the breather oscillates at the natural frequency ($\Omega_b = 1.0$) of the modified sine-Gordon equation.

In Fig.1 we show a typical quasiperiodic solution which can be observed for a variety of parameters. Since only one external drive is present, the system itself must be the origin of the second frequency incommensurate

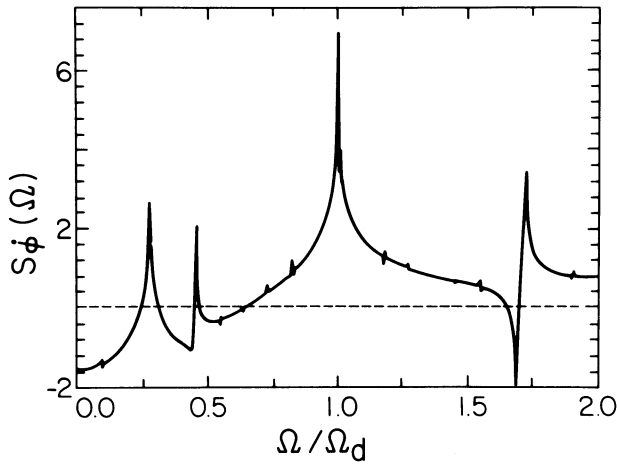


Fig. 2. Voltage power spectrum $S_{\phi_t}(\Omega)$ vs. (Ω/Ω_d) higher peaks corresponds to the drive and breather frequencies.

with the external drive. In Fig.2 we show the power spectrum as a function of frequency for the same parameters as in Fig.1. Two peaks are clearly visible at $(\Omega/\Omega_d)=1.0$ and $\approx\sqrt{3}$. The first one obviously corresponding to the rf drive frequency. The second corresponds to the frequency of oscillation of the breather-like excitation. Then, the origin of the quasiperiodicity is clear: it arises from the irrational relationship between the forcing frequency and the oscillation of the spatial mode, the latter being a result of the external ac bias applied to the system. Thus, once again the Josephson junction exhibits behavior quite analogous to other dynamical systems. In this case, the observed behavior is quite similar to that of Rayleigh-Bénard convection, where an oscillating drive is applied either through an oscillating temperature difference (6) or through an oscillating external magnetic field (7), the temperature difference acting in both cases as the equivalent of our magnetic field. The main difference between the two cases arises from the fact that the spatial oscillation in the Josephson junction is of

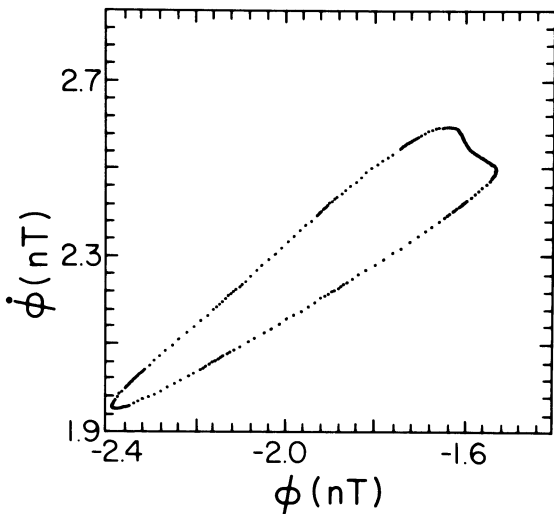


Fig. 3. Poincaré map $\phi_t(nT)$ vs. $\phi(nT)$: note variation in the density of points.

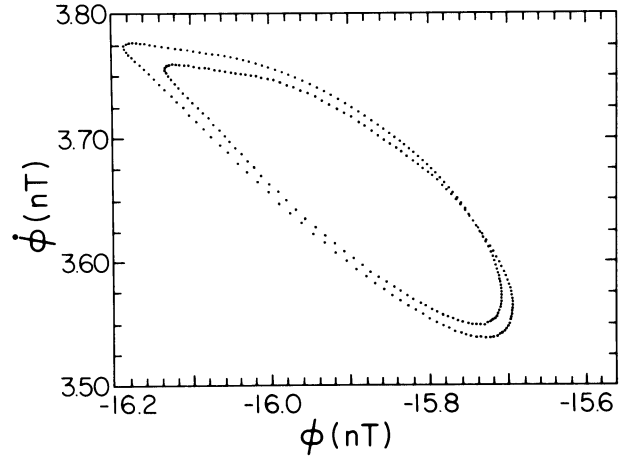


Fig. 4. Poincaré map $\phi_t(nT)$ vs. $\phi(nT)$: doubling of the torus.

solitonic character. Work is in progress to determine the universality of this transition by the study of its multifractal character.

If the drive is increased further, it is possible to observe a variety of phenomena. Subharmonic response at the driving frequency produces regions of the quasiperiodic attractor visited more frequently than others and eventually a periodic window appears; this effect is shown in progress in the Poincaré map of Fig.3; an experimental map analogous to this portrait has been reported for the forced Rayleigh-Bénard system (7). Occasionally, the quasiperiodicity can bifurcate due to the generation of subharmonics at the breather frequency while the system remains quasiperiodic. This doubling of the torus can be seen quite clearly in the Poincaré map of Fig. 4.

4. CONCLUSIONS

We have shown that quasiperiodicity can be autonomously excited in a long Josephson junction with a single oscillating drive by the interaction between the spatial and temporal modes of the system. This quasiperiodic behavior is quite analogous to that of higher dimensional systems such as Rayleigh-Bénard convection and presents evidence for the universality of the Q_2 to chaos transition in dynamical systems.

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