Physica B 194–196 (1994) 411–412
North-Holland

PHYSICA B

Roughening transition in a thermal sine-Gordon system

R. E. Rangel, L. E. Guerrero and A. Hasmy.

Centro de Física, Instituto Venezolano de Investigaciones Científicas,
Apartado 21827, Caracas 1020 A, Venezuela.

We relate the appearance of noise induced solitons in a sine-Gordon system with the roughening exponent, defined as the scaling exponent of the length of the ensemble average of the standard deviation of the height of the spatiotemporal profile. We find that before the onset of the noise-induced transition to the solitonic regime, the roughening exponent is zero as would correspond for a white noise signal. After the activation of solitons this exponent exhibits a crossover from ~ 0.70 to ~ 0.50. We point out the connection of our results to models for surface growth and random deposition, particularly, the stochastic Kardar-Parisi-Zhang model.

In many physical systems like for example crystal growth [1] and two phase flow in porous media [2-3] the dynamics and geometry of surfaces is attracting an enormous interest.

In this paper we characterize geometrically the appearance of noisy activated solitons in a sine-Gordon model [4]. We show that this system exhibits two different self-affine regimes after the onset of the solitonic regime. We also relate our results with the Kardar-Parisi-Zhang equation (KPZ) or random Burgers equation, which has been very successful in explaining a broad range of structures generated by non-equilibrium stochastic processes [5].

The random and damped sine-Gordon model describes nucleation-dominated crystal growth [6] if one considers the solution $\phi(x,t)$ as the height of a one dimensional surface. The considered model is given by the following dimensionless equation:

$$\phi_{xx} - \phi_{tt} - \sin\phi = \alpha\phi_t - R(x,t) \qquad (1)$$

In our simulations the noise term $R(x,t)$ is uncorrelated both in time and space. We use flat initial conditions ($\phi(x,0) = \phi_t(x,0) = 0$) and open boundary conditions ($\phi_x(0,t) = \phi_x(l,t) = 0$). In all the results to be presented in this article, $\alpha = 0.252$ and $l = 160$; we discretize the equation into 4096 points.

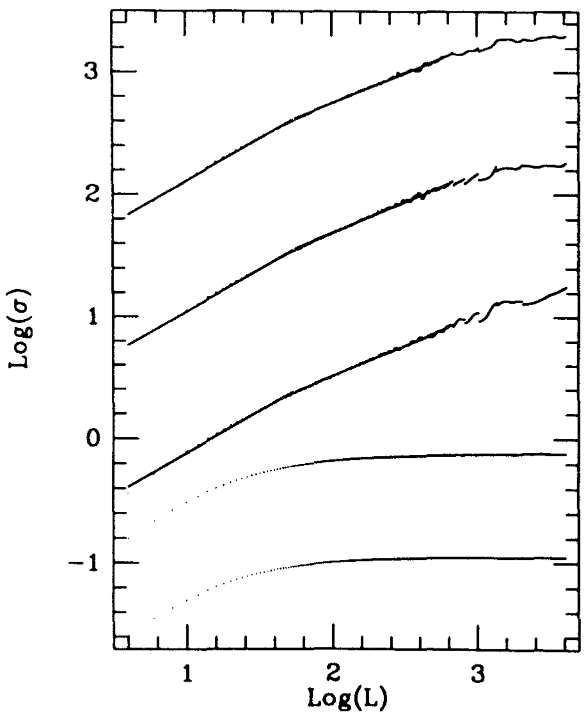

Figure 1. $ln\,\sigma$ vs. $ln\,L$ for increasing values of the variance of the noise. We present for the curves corresponding to solitonic regimes linear fits with slopes ~ 0.70 and ~ 0.50.

Figure 1 shows a log-log plot of $\sigma(L)$ as a function of L, where $\sigma(L)$ is the ensemble average of the standard deviation of the spatiotemporal profile as a function of the length scale L, i.e.,

SSDI 0921-4526(93)E0785-F

$$\sigma(L)=\left\langle\left[\frac{1}{N_L}\sum_{i=1}^{N_L}(\phi_i-\bar{\phi})^2\right]^{1/2}\right\rangle;\bar{\phi}=\frac{1}{N_L}\sum_{i=1}^{N_L}\phi_i \quad (2)$$

Here $\langle\ldots\rangle$ means ensemble average. In our simulations we average over 5000 realizations for each size L sampled once the stationary regime has settled down, in such a way that $\sigma(t,L)=\sigma(L)$. We obtain different ensembles by taking the surface $\phi(x,t)$ separated by a long enough time interval and by dividing the total length l in segments of size L. In Eq. (2) the index i run over the number of points N_L contained in the segment of length L.

The surface $\phi(x,t)$ is a random or stochastic function being the solution of a nonlinear evolution equation with noise; if $\phi(x,t)$ is a self-affine function we expect $\sigma(L)$ to scale as $\sigma(L)\propto L^\zeta$ [7], where ζ is the Hurst exponent [2], which is called in the case of surfaces the roughening exponent. This is confirmed in Figure 1 that shows the expected scaling behavior for five different variances of the noise (from below 1.7, 10.0, 20.0, 283.3 and 3,333.3). The third curve is around the dynamic transition to the ordered state with a reduced number of degrees of freedom [4].

The roughening exponent ζ displays beneath the transition (the pair of lower curves) the behavior characteristic of white noise, i.e., $\zeta=0$. Above the transition (upper curves) the roughening exponent exhibits two different scaling behaviors revealing a crossover behavior: for small length scales $\zeta\sim 0.70$ while for larger length scales, $\zeta\sim 0.50$.

We now relate our results with the KPZ model [5], which is given by the equation,

$$\frac{\partial h}{\partial t}=\nu\frac{\partial^2 h}{\partial x^2}+\frac{\lambda}{2}\left(\frac{\partial h}{\partial x}\right)^2+R(x,t) \quad (3)$$

Here $h(x.t)$ is a surface which behavior obeys the relation $\sigma_{KPZ}(t,L)=L^\zeta f\left(t/L^z\right)$, with $f(x)\rightarrow const$ as $x\rightarrow\infty$ and $f(x)\sim x^{\zeta/z}$ for small x; for $\lambda\neq 0$, $\zeta=1/2$ and $z=3/2$, and for $\lambda=0$, $\zeta=1/2$ and $z=2$. For $t>>L^z$ the stationary regime is achieved and $\sigma_{KPZ}(t,L)=\sigma_{KPZ}(L)\propto L^\zeta$, whereas for $t<<L^z$, $\sigma_{KPZ}(t,L)=\sigma_{KPZ}(t)\propto L^{\zeta/z}$.

In our results we find above the onset of solitons a very interesting crossover from non-KPZ behavior ($\zeta\sim 0.70$) to KPZ behavior ($\zeta\sim 0.50$). Such a kind of roughening crossover is found for instance in models of displacement in porous media [3]; besides this, continuum models of crystal growth considered by Villain [1] also show a non-KPZ behavior at small length scales.

Considering the picture of solitons and antisolitons being excited randomly in a random sine-Gordon system [8], we interpret tentatively our results as an effective coherence that appears above the transition to the ordered state. The two different self-affine regimes after the dynamic transition should be related with two different coherence strengths. This shows itself in a stronger roughening exponent at small length scales. Finally it appears that for sufficiently large scales, the solitonic coherence ceases and a crossover to a zero roughening exponent takes place (see upper pair of curves in Figure 1).

Summarizing, we have shown quantitatively that the coherence of the ordered state appears to have three different regimes with well-defined crossover lengths. Further investigation is in progress to precise the dynamic exponent ζ/z as well as the meaning of the crossover lengths we have found.

REFERENCES

1. J. Villain, J. Phys. I 1 (1991) 19; P. Meakin, R. C. Ball, P. Ramanlal and L. Sander, Phys. Rev. A 35 (1987) 5233.
2. F. Family and V. Vicsek (eds.), Dynamics of Fractal Surfaces, World Scientific, Singapore, 1991.
3. R. Rangel and J. Rivero, Physica A 191 (1992) 253.
4. L. E. Guerrero, A. Hasmy and G. J. Mata, in these proceedings.
5. M. Kardar, G. Parisi and Y.-C. Zhang, Phys. Rev. Lett. 56 (1986) 889; E. Medina, T. Hwa and M. Kardar, Phys. Rev. A 39 (1989) 3053.
6. J. Krug. and H. Spohn, Europhys. Lett. 8 (1989) 219.
7. B. Mandelbrot, Physica Scripta 32 (1985) 257.
8. M. Büttiker and R. Landauer, in: Nonlinear Phenomena at Phase Transitions and Instabilities, T. Riste (ed.), Plenum, New York, 1982.