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Turbulence in Josephson junctions

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We show that a solid-state device, the long Josephson junction, behaves very much like other turbulent systems exhibiting regimes of weak and hard turbulence. Because of the discrete nature of our simulations, we conclude that while a large number of active degrees of freedom are necessary for the existence of turbulence, their number can indeed be small. We show that the transition to the turbulent state is intimately related to the spatiotemporal symmetry breaking and to the existence and breakdown of spontaneous pattern formation in the system.

The remarkable interest in chaotic phenomena in dynamical systems during the last decade arose in part by the tantalizing perspective that turbulent systems could be understood within the framework of temporal chaos in low-dimensional systems.[1] While it is true that most hydrodynamical turbulent systems such as Rayleigh-Bénard convection exhibit most of the characteristic routes to chaos associated with low-dimensional systems,[2] it has also become quite clear that as such systems are driven harder; this simple characterization is no longer valid. The question is then how to characterize the various regimes in the system as the "turbulent" behavior departs from the better understood "chaotic" behavior, as well as establishing what determines the ability of a system to become turbulent. While such a characterization does not exist, it seems evident that a temporal description will not suffice and that spatial effects in systems with a large number of degrees of freedom need a description of its own.

While it is commonplace to talk about turbulence only in systems with a large number of degrees of freedom, systems with a small (but not necessarily low) number of degrees of freedom can also exhibit a variety of interesting spatiotemporal effects such as spontaneous pattern formation and temporal chaotic behavior. The question then arises about whether systems with a small number of degrees of freedom may also exhibit behavior which can be characterized more appropriately as turbulent, how this transition takes place, and how it may be characterized. In this paper, we suggest that driven, long Josephson junctions (LJJ) in fact behave very much like turbulent systems as characterized by the absence of spatial correlation for a wide range of frequencies. Furthermore, we show that the system approaches this turbulent state through a well-defined transition beyond the temporal chaotic state which is intimately related to the existence and breakdown of spontaneous pattern formation in the system. These dynamics seem more like chaotic behavior associated with space rather than time. Due to the discrete nature of our simulations, we conclude that while a large number of active degrees of freedom are necessary for the existence of turbulence in a system, the number can indeed be small. In fact, our system is isomorphic to a chain of 128 coupled damped pendula. This analogy indicates that such turbulent behavior is indeed ubiquitous in nature as it can be present not only in fluids but also in solid-state and mechanical systems.

The forced, long Josephson junction is described approximately by a sine-Gordon-like equation given by

$$-\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial t^2}+\frac{1}{\sqrt{\beta_c}}\frac{\partial\phi}{\partial t}+\sin\phi=\rho\sin(\Omega_d t)\ , \qquad (1)$$

where $\phi=\phi(x,t)$ is the phase difference of the superconducting order parameter between each side of the barrier and β_c is the Stewart-McCumber parameter (a measure of the damping of the system); the distance is normalized to the Josephson penetration depth λ_J, time is normalized to the inverse of the Josephson plasma frequency, the rf amplitude ρ is normalized to the critical current I_c, and Ω_d is the normalized applied frequency. The external applied field is taken into account through

$$\partial\phi(0,t)/\partial x=\partial\phi(L,t)/\partial x=\eta\ ,$$

where L is the junction length and η is a measure of the external magnetic field. In contrast with previous approaches[3,4] in which finite boundary conditions introduce a spatial symmetry breaking that induces the pattern formation, in this paper, unless otherwise stated, we employ open boundary conditions $\eta=0$ and initially $\phi(x,0)=\partial\phi(x,0)/\partial t=0$ so that the pattern formation and convertion phenomena we observe can be regarded as fully spontaneous. In addition we use $L=5\lambda_J$ or $10\lambda_J$, $\beta_c=15.744$, and $\Omega_d=0.65$ which are realistic parameters.

Consider first a junction with $L=5\lambda_J$. At low rf drives ($\rho<1.75$), such a junction behaves very much like a Josephson junction without spatial extent in that the chaotic regime is reached through the usual bifurcation route with all solutions being spatially homogeneous. In contrast to this low-amplitude regime, a further increase of the bias amplitude introduces collective effects which lead to pattern formation, turbulence, and novel spatiotemporal phenomena.

When the rf drive is increased beyond $\rho=2.5$ the system goes into an intermittent state between two period 1 states and chaos as shown in Fig. 1(a), where we show a local stroboscopic plot of the voltage at each period of the rf drive; while this intermittency is reminiscent of the noise-induced hopping that has been observed in simulations of small junctions,[5] this is in fact much different since instead of being related to jumps between unstable attractors in phase space it is due to the inability of the system to sustain a steady-state spatial pattern of the breather type. This can be demonstrated by looking, for example, at Fig. 1(b) where the phase difference between two different points of the junction is illustrated. Each of the intermittentlike jumps in Fig. 1(a) is associated with a sudden loss of phase coherence across the junction. Figure 1(c) demonstrates this behavior where one sees that during part of the cycle the system forms a breather but is unable to sustain it. After this breakup of phase coherence the system might still be in the chaotic state but the spatial pattern remains homogeneous in phase space where it stays for what can be considerably long periods of time. Each time the system attempts to form a breather, however, it collapses breaking phase coherence. Thus, it is the spatial variable which is responsible for the novel behavior, as it is the tendency of the system to spontaneously form a spatial pattern that leads to what can be considered, in some sense, to be a spatial intermittency without the spatial symmetry of the system being broken.

If the junction is somewhat larger the interaction between the degrees of freedom is decreased (in the analogy of coupled pendula of Eq. (1) we are decreasing the coupling between the pendula). This favors the spontaneous symmetry breaking of the system, both in space and in time. For a LJJ with $L = 10\lambda_J$ and $\rho = 2$ we find a rich transition from a transient with period 1 to a period 2 regime which at first sight might appear to be a simple bifurcation in time as in low-dimensional dynamical systems. However, as shown in Fig. 2(a) where we plot the

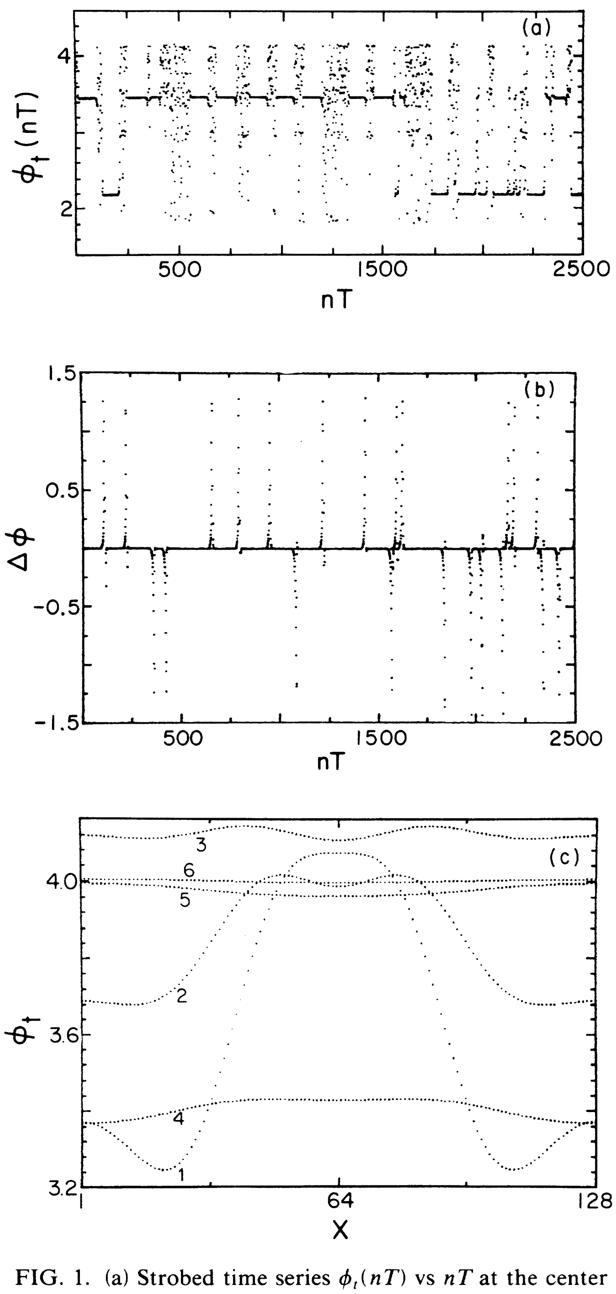

FIG. 1. (a) Strobed time series $\phi_t(nT)$ vs nT at the center of the junction showing intermittency; (b) Difference of the phase between two points of the barrier $\Delta\phi(nT)$ vs nT revealing loss and recovery of the homogeneous character of the solution; (c) Strobed profile $\phi_t(x,nT)$ vs x—breather decay into a spatially homogeneous state.

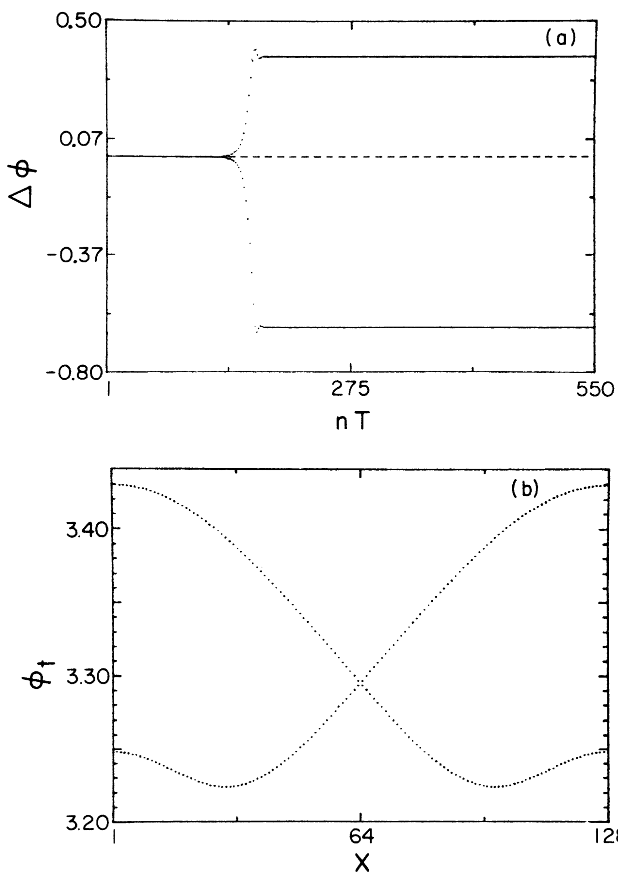

FIG. 2. Spontaneous symmetry breaking. (a) Difference of the phase between two points of the barrier $\Delta\phi(nT)$ vs nT. (b) Strobed profile $\phi_t(x,nT)$ vs x showing nonsymmetric breather oscillation.

phase difference $\Delta\phi$ between the two points of the junction for each period of the rf drive as a function of time, the voltage is no longer homogeneous in space. The fact that $\Delta\phi$ repeats only every second period shows that the spatial extent of the junction has now become significant. Figure 2(b) indicate that the breather present at lower drives is no longer stable but instead a "virtual" breather centered at the edge of the junction is formed with only half of it present inside the junction. This "virtual" breather switches at each period between its two states, thus yielding a steady state in which the spatial symmetry is broken, muck like the symmetry-breaking precursor (in phase space) of the usual transition to chaos. Figure 2(b) can be interpreted to be the inability of the system to sustain a soliton commensurate with its size jumping instead into a sort of bifurcation in space. The system thus succeeds in breaking the symmetry in real space and not in phase space as in temporal chaos.

Up to now in this paper we have described novel dynamics which the LJJ exhibited which are associated with the competition and switching between collective dynamics and low-dimensional or single-particle dynamics. In what follows we show a transition which can be characterized as spatiotemporal disorder.

Previous results[4,6] have shown that sine-Gordon-like systems, and in particular the LJJ, exhibit a transition to chaos associated with the spatial extent of the junction which is in some sense similar to chaotic dynamics in low-dimensional dynamical systems. The LJJ can also present transitions to chaos in the usual temporal sense: indeed, we found strange attractors corresponding to homogeneous regimes. In both cases the system reaches a chaotic state which can be rather well characterized by a self-similar strange attractor with a well-defined fractal dimension.

For the same junction length used previously ($L=10\lambda_J$) the system goes into a disordered state in which the attractor completely loses its fractality and autosimilarity. An example of this is shown in Fig. 3(a) where we present the Poincaré map for a low enough number of periods to show the dense formation of this disordered state ($\rho=2.5$). The corresponding return map for each period of the rf drive for the same parameters exhibits a complete absence of structure. These plots reveal that low-dimensional strange attractors are destroyed due to an activation of an increased number of effective degrees of freedom. This solution is inhomogeneous in space and also exhibits symmetry breaking; solutions are no longer coherent in space and the exchange of energy between the degrees of freedom is disordered. This disordered state is clearly quite different from that of low-dimensional chaos; the question arises as to how can one characterize it and whether it bears any relationship to the turbulent behavior seen in fluid systems. We find that the fractal measure of the Poincaré map for this attractor for the whole junction or for a single point of the junction tends to the Euclidean value. We have chosen to study this behavior in the context of the soft and hard turbulent states identified by Heslot *et al.* in Rayleigh-Bénard convection.

In order to further characterize any disordered state in a spatiotemporal system one has to resort to studying possible correlations in space for various frequencies which can be done through the study of its coherence. The coherence is defined as

$$C_{xy}(k)=\frac{|G_{xy}(k)|^2}{G_{xx}(k)G_{yy}(k)}\ , \tag{2}$$

where G_{xx} and G_{yy} are the autopower spectra of the two real sequences and G_{xy} is the cross-power spectrum. The absence of coherence at any frequency shows that there are no correlations between the temporal dynamics at two points of the system. In fact, the absence of these correlations indicates that the construction of higher-dimensional phase spaces using more of the spatial points of the junctions results in little information of value.

Figure 3(b) shows the calculation of the coherence for the dynamical state of Fig. 3(a). The coherence between two points (the middle and an extreme of the LJJ) disappears for all values but those of very low frequencies and the driving frequency. This is quite similar to the results of Heslot, Castaing, and Libchaber for Rayleigh-Bénard convection,[3] who named this regime the soft-turbulence

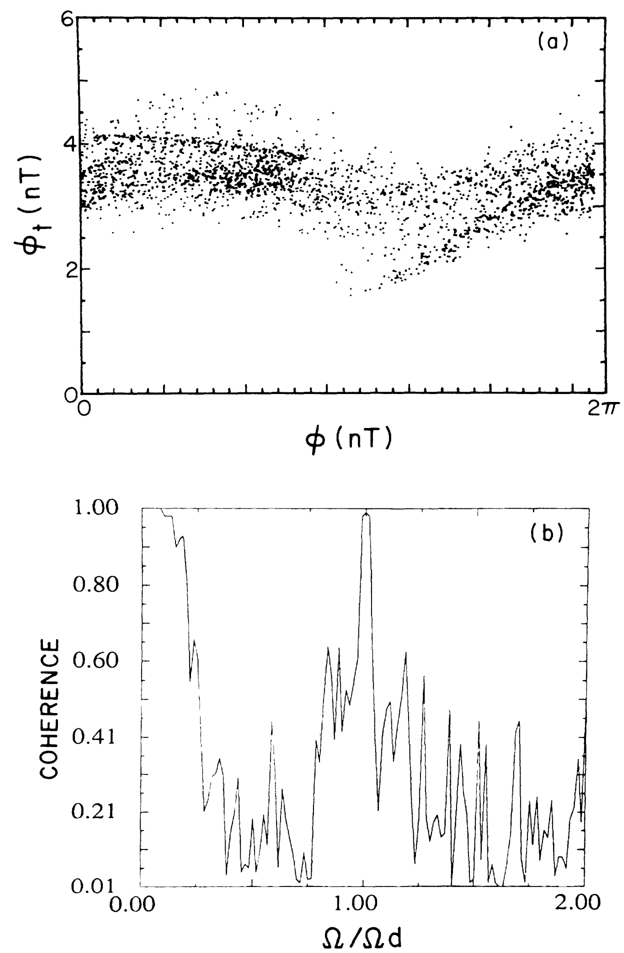

FIG. 3. Destruction of the strange attractor. (a) Poincaré map [mod(2π)]—attractor loses fractality; (b) Coherence revealing the soft-turbulencelike regime.

regime—a regime disordered both in space and in time. Our results show that the soft-turbulence regime might be quite generic of the transition to turbulence in various regimes and that this transition might not be limited to systems with a large number of degrees of freedom. Note that the two systems differ not only in their number of degrees of freedom, but also in the Rayleigh-Bénard system, symmetry breaking is induced via boundary conditions which induces oscillations while the LJJ is driven by an oscillation. In addition, our results were obtained in the absence of thermal noise which is unavoidable in experiments. In this sense it is clear that these turbulent states can be regarded as being deterministic in origin.

One of the limitations of solving Eq. (1) is that it is difficult to explore solutions in parameter space in detail due to the long computational times. While we have not found a hard-turbulence regime near that of the soft-turbulence regime described previously, we have been able to find a hard-turbulence regime analogous to the one described by Heslot *et al.*[3] for the Rayleigh-Bénard system, but for a junction with $L=5\lambda_J$ and $\rho=0.7375$ and with a magnetic field present ($\eta=1.25$). In this regime the pattern formation and conversion[4] induces a reemergence of coherence, but only in a narrow frequency range in the form of a peak at a certain frequency as shown in Fig. 4. (There is also a peak at the driving frequency, absent in Rayleigh-Bénard convection since there is no oscillating driving force.) This indicates that the system dynamics are only correlated in space at a single frequency. Thus, the behavior of the LJJ can be considered to be turbulent in the same sense as it has been possible to characterize the transition to turbulence in fluid systems through the identification of soft- and hard-turbulence regimes above the chaotic transition. This represents a strong suggestion of universality in the transition turbulence.

One can look further into the nature of this transition by looking at the histogram of fluctuations in both regimes as described by Helsot *et al.*[3] Our simulations also show differences between soft and hard turbulence. However, they are also different from those reported by Helsot *et al.*[3] suggesting that this might not provide a clear-cut distinction between these two regimes.

Our simulations have shown that the dynamics of the LJJ can be considered to be turbulent suggesting that the description of the transition from chaos to turbulence via two different turbulent regimes is generic to systems with a wide range in the number of degrees of freedom. While it might be feasible to perform experiments with LJJ's by placing contacts over various places of the junction and measuring correlation in time and space, the high frequency oscillations of the LJJ present a limit to distinguishing the difference between the soft- and hard-turbulence regime since only low-frequency behavior well below the driving frequency can be measured.[7] This suggests that it might be more interesting to look for similar behavior in solid-state systems where such a difference can be distinguished such as solids exhibiting charge-density waves or even two-dimensional arrays of Josephson junctions drivein at low frequencies.

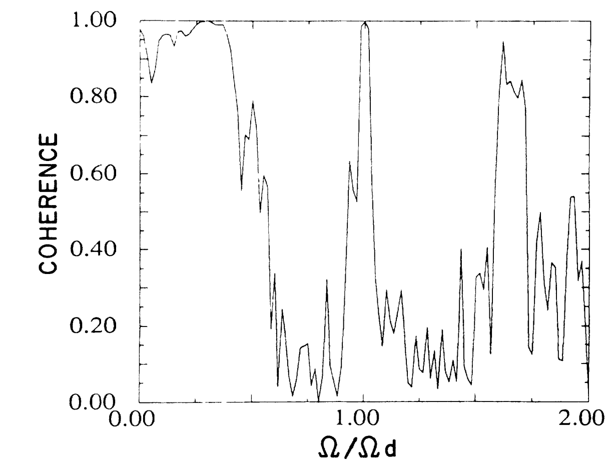

FIG. 4. Chaotic pattern formation and conversion. Coherence reveals a hard-turbulencelike regime.

In conclusion we have shown that LJJ exhibits unique and novel dynamics associated to spatiotemporal behavior and to symmetry breaking both in phase and real spaces. In addition, we have shown that in such a system the transition from periodic to chaotic and to turbulent dynamics appears to be quite general in nature, exhibiting two separate regimes of soft and hard turbulence as characterized by their spatial coherence. Other characterizations of these complex spatiotemporal regimes in dynamical systems might provide a universal common picture for the transition to turbulence in a wide variety of systems.

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