Coupled Josephson Soliton Oscillators

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Abstract. We report on phase-locking phenomena in systems of two closely spaced quasi-one-dimensional Josephson transmission lines, JTLs, biased in resonant fluxon modes where the junctions work as soliton oscillators with a fundamental frequency of 10 GHz. The coupling which is *broad-band* is seen both in the I-V curves and in the microwave signals from the JTLs. Enhanced microwave emission is observed experimentally. A theoretical model for a magnetic dipole coupling mechanism is proposed. Numerical simulations based on the model reproduce the phase-locking phenomena but not the enhanced emission.

1. Introduction

Recently, the non-linear behavior of long and narrow, quasi-one-dimensional Josephson tunnel junctions has been attracting much attention. The junctions act as non-linear Josephson transmission lines (JTLs) in which high-Q resonance modes may be excited by the internal Josephson oscillation [1]. The present study was aimed at achieving coherent radiation from pairs of coupled JTLs. Phase-locked arrays of Josephson soliton oscillators are of technological interest due to their larger power and smaller linewidth [2].

For a narrow junction of length L longer than the Josephson penetration depth λ_J and width $W < \lambda_J$, the travelling wave on the line has soliton character. The soliton/antisoliton is a $\pm 2\pi$ kink/antikink in the Josephson phase, ϕ, across the tunnel barrier between the two superconducting electrodes of the junction.

There is a quantum of magnetic flux, a fluxon/antifluxon, associated with the phase kink/antikink. The equation of motion for the Josephson phase is the much studied perturbed sine-Gordon equation, see [3].

The Lorentz force exerted on the fluxon/antifluxon by a bias current through the junction accelerates the soliton until it reaches a steady-state condition where the total loss balances the input power from the bias. The maximum velocity of the soliton is $u_\infty = \bar{c}$, the Swihart velocity of light in the barrier (for Nb-Nb$_x$O$_y$-Pb junctions, $\bar{c} = 0.025\ c_{vacuo}$). The motion generates a maximum voltage of $V = -d\Phi/dt = -2\Phi_0 u_\infty/2L = -\Phi_0 \bar{c}/L$. For a junction of length 400 μm this

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corresponds to a voltage around 40 µV giving a Josephson frequency of about 20 GHz. If N fluxons/antifluxons are threading the barrier, the voltage of the *resonant zero-field steps*, ZFSs, will be proportionally larger, $V_{ZFS\pm N} = \mp N\Phi_0\bar{c}/L$.

Another type of steps, Fiske steps (FS) are observed when the boundary conditions are determined by an applied magnetic field. They correspond to alternate motion of fluxons and plasma waves along the junction barrier (fluxons are created in one end of the JTL and annihilated in the other end). In the first Fiske mode, on the average only half a fluxon is present per cycle. On the Mth Fiske step the dc voltage is $V_{FS\pm M} = \mp M\Phi_0\bar{c}/2L$.

When a soliton arrives at the end of the JTL it is reflected and a fraction of the energy in the fluxon is emitted as a pulse. The frequency detected at *one* end of the JTL is half of the Josephson frequency, $f = f_J/2 = eV/h$, in the case of a 400 µm long junctions f is about 10 GHz.

In the following we report on one way of coupling two or more of the non-linear JTLs together in order to study the interaction between them and to increase the output power level.

As a measure of the in-phase coherence between two locked oscillators we use the enhancement factor, F, defined as $F = (P_{12} - P_1 - P_2)/2\sqrt{P_1 P_2}$ which has a maximum value of 1 for two point-like oscillators running totally in-phase. When F exceeds 1 we say that the system exhibits enhanced emission of radiation.

2. Experimental Technique

Figure 1 shows the sample geometry with two long and narrow, 400×20 µm^2 overlap Nb-Nb$_x$O$_y$-Pb tunnel junctions spaced by 35 or 75 µm and sharing the Pb top

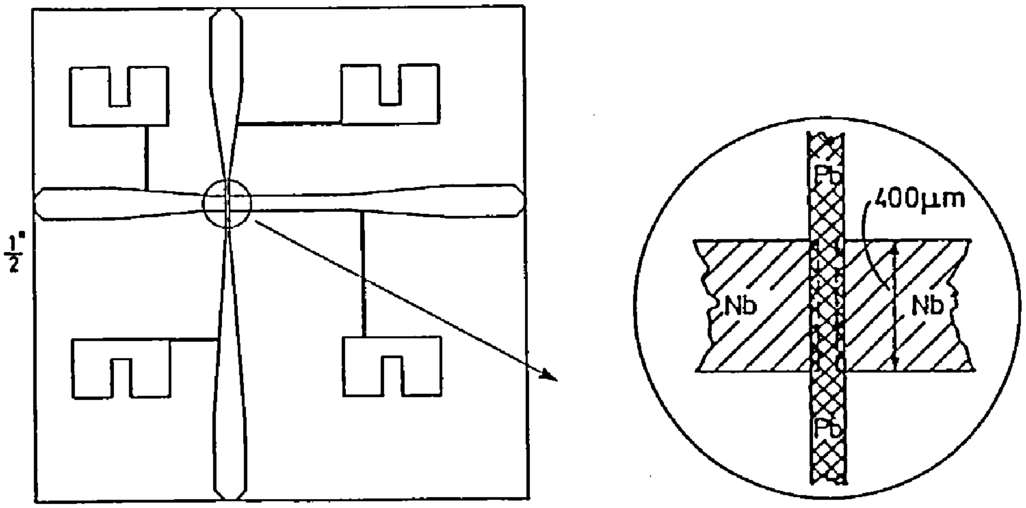

Figure 1. Sample geometry: two long and narrow Nb-Nb$_x$O$_y$-Pb junctions of the overlap type form two Josephson transmissions lines (JTLs) which share a common Pb top electrode.

electrode. The substrates were 1"×1" 0.5 mm thick Corning 7059 glass with low microwave loss. Typical junction parameters were: critical current density $J_c = 3$ - $16\ \mathrm{A/cm}^2$, $\lambda_J = 100$ - $250\ \mu\mathrm{m}$ (i.e., $L/\lambda_J = 4$ - 1.6), and normalized Swihart velocity $\bar{c}/c = 0.025$. Two independent current sources were used: *a) either* to bias the two JTLs individually *or b)* to bias both of them with one current source (I_{COMMON}) using the second source to provide an additional current (I_{TRIM}). The total voltage across the two junctions was measured. Two current configurations were used: *series aiding* and *series opposed*, given by the relative directions of the two bias currents. In the following the notation +/+ (-/-) and +/- (-/+) will be used to denote the two configurations. For further details about the experimental set-up we refer to Ref. [4].

3. Experimental Results

The experimental results reported here are based on measurements on four samples. The dc I-V curves and the microwave spectra were recorded for the ZFSs and the FSs for the JTLs individually and for both of them running at the same time.

In Fig. 2 we present a typical observation of the frequency-pulling, -mixing and -locking behavior observed when two soliton oscillators both biased in the ZFS1

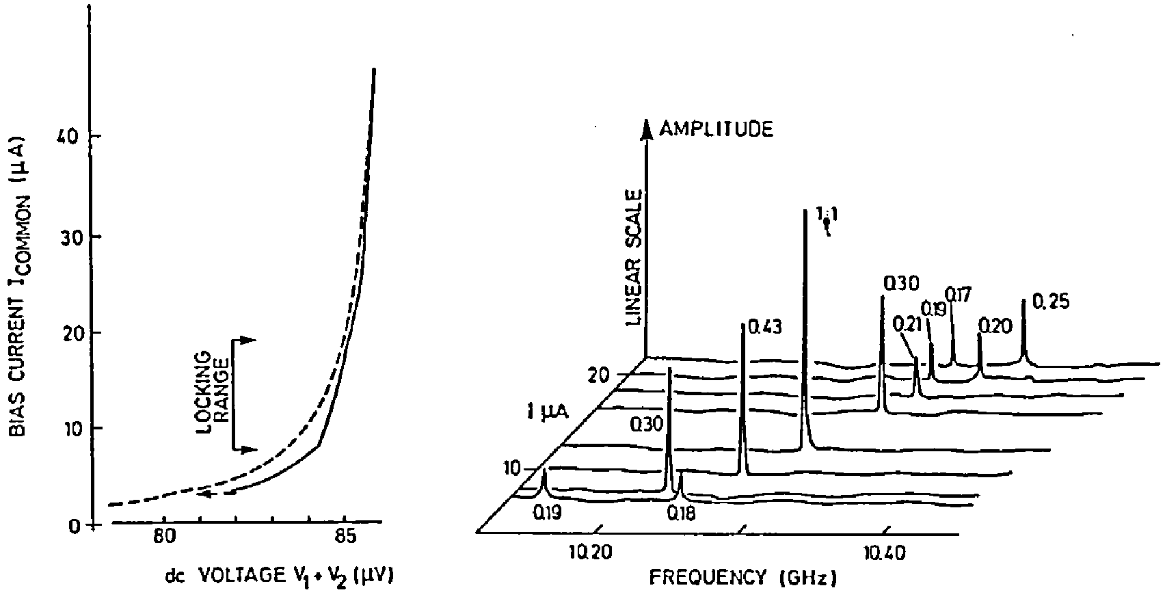

Figure 2. Experimental observation of phase locking, both in the I-V curve and in the microwave spectra, between two JTLs biased in the series aiding mode ZFS+1/ZFS+1 at 4.2 K (S-6006/2). Voltage-pulling and -locking were observed when a trim current, I_{TRIM}, was applied to one of the junctions (to compensate for the difference in junction parameters). The full curve was recorded for $I_{TRIM} = 23$ μA, the dashed curve for $I_{TRIM} = 0$. The corresponding microwave spectra are shown as voltage amplitude vs. frequency for a number of bias points inside and outside the locking range. The total integrated power received is indicated for each peak.

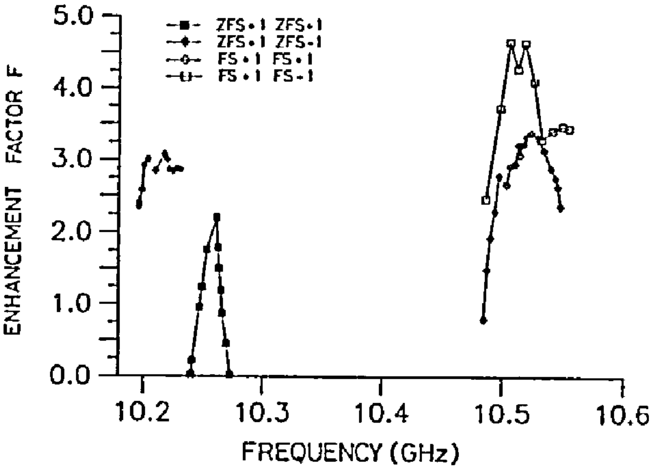

Figure 3. Enhancement factor, $F = (P_{12} - P_1 - P_2)/2\sqrt{P_1 P_2}$ vs. frequency over locking ranges for four different values of bias currents (I_2 swept, I_1 constant), chosen to give phase-locking for all four possible combinations of ZFS±1 and FS±1 in the two Josephson soliton oscillators (sample S-6006/2).

mode were tuned into a phase-locked state. The "3D" plot shows the spectra of detected voltage (linear scale) vs. frequency taken for different values of the common bias current ($I_{TRIM} = 23\,\mu A$). The integrated power is indicated for each spectrum. It is calculated by multiplying the peak *power* by the *full linewidth at half maximum power (FWHM)*.

From the measured values of P_{12}, P_1 and P_2 the enhancement factor F was found. By sweeping through the locking range by varying the current through one JTL while keeping the other one constant, the variation of F over the locking region was determined. Figure 3 shows such a measurement of the enhancement factor F vs. frequency. The most surprising observation was that F in the middle of the locking region exceeded one by up to about a factor of five. We note that there was a tendency toward stronger interaction (i.e., stronger frequency-pulling) and also higher values of F for the case of two interacting Fiske modes than for interacting zero-field step modes. The emission enhancement was stronger in the series opposed current configurations, ZFS+1/ZFS-1 and FS+1/FS-1, than in the series aiding ones, ZFS+1/ZFS+1 and FS+1/FS+1. The coupling phenomena were observed over all overlapping voltage ranges and for all combinations of steps, e.g., ZFS+2/ZFS+1.

4. Theoretical Model and Numerical Simulations

A number of theoretical models have been proposed and explored for the interaction between Josephson transmission lines, based on capacitive, inductive or resonant coupling [5]. Provided realistic parameters are used, numerical simulations predict phase-locking phenomena for all these coupling models.

For our geometry with two closely spaced, parallel overlap junctions, the model based on the equivalent circuit with broad-band, mutual inductive coupling seems most appropriate. It was investigated in Ref. [4]. Numerical simulations of the coupled sine-Gordon equations reproduced the experimentally observed features of the current-voltage characteristics, most notably the locking regions.

Using the inductive coupling model we have investigated possible origins of the observed large enhancement factors by computing the time-dependent voltages at both ends of the long junctions. The important point to note is that F never exceeds 1, in contrast with experimental findings.

In an attempt to resolve this discrepancy, we have investigated the effect of non-linear damping on the coupling. A two-segment model for the quasi-particle I-V characteristics of the Josephson junction elements was used, the dissipation coefficient α thus depended upon the local instantaneous value of ϕ_t. These simulations also generated maximum F values of 1.

5. Discussion and Conclusions

The experimental observations of enhanced emission ($F > 1$) remains unexplained and the source of this effect is unknown. One possible important mechanism which is yet to be investigated concerns the dynamics of fluxon flipping at the ends of the junctions. It is generally assumed that the radiation originates in this specific stage of the fluxon motion. Simply averaging ϕ_t at the junction boundaries ($x = 0, L$) may not take into account the important acceleration and decelerations which are occurring during reflection. In the frequency regime, such changes in the soliton phase and waveform correspond to conversion of power between harmonics in the soliton spectrum. In the experiments only the power emitted at the fundamental frequency of the fluxon motion (around 10 GHz) is detected. Future experiments will be aimed at capturing more information about the pulse-like microwave emission from single and coupled Josephson soliton oscillators.

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